ON ORDERINGS JAROSLAV NEŠETŘIL CHARLES UNIVERSITY PRAGUE

LES HOUCHES 28/5/25

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JOINED WORK WITH GABOR KUN (RENYI INST.)

JAROSLAV NEŠETRIL CHARLES UNIVERSITY PRAGUE

on orderings



3 THMS 2 CONJECTURES

15' TOTAL

GIVEN G DOES THERE EXISTS AN ORDERING < OF VERTICES OF G SUCH THAT F[<] FAILS TO BE A SUBBRAPH OF G[<]?



FOR EXISTENCE OF ORDERING

DECISION PROBLEM

G HAS AN ORDERING <such that $P_n \notin G^{<}$ \Re $\chi(G) \leq n$

G HAS AN ORDERING < SUCH THAT P. ⊈ G[<] T $x(G) \leq n$ GALLAI, HASSE, MINTY, ROY, VITAVER (MATJASE VICH)

MORE GENERALLY

- F ORDERING PROBLEM
- FOR A FINITE SET 54
 - OF ORDERED GRAPHS.



OF ORDERED GRAPHS.

FOR A FINITE SET 5

MORE GENERALLY

QUESTIONS

ARE COLORING AND ORDERING PROBLEMS POLYNOMIALLY EQUIVALENT?

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ARE COLORING AND ORDERING PROBLEMS POLYNOMIALLY EQUIVALENT?



IS THERE DICHOTOMY

(HELL, MOHAR, RAFIEY 14)

FOR ORDERING PROBLEMS?

FEDER-VARDI CONJECTURE PROVED BY BULATOV ZHUK ALGEBRAIC METHOD (JEAVONS ET AL.)

COLORING PROBLEMS (H-COLORING) HAVE DICHOTOMY (I.E. EVERY H-COLORING PROBLEM IS EITHER P OR NP-COMPLETE

THMI (G.KUN, J.N. 25)

THERE IS NO DICHOTOMY FOR 5 - ORDERING PROBLEMS

(ASSUMING PANP)

- "FULL POWER OF ORDERING "
- THERE ENTRY AND F- ORDERING SUCH THAT L AND F- ORDERING PROBLEM ARE POLYNOMIALLY EQUIVALENT.
- FOR EVERY LANGUAGE LE NP THERE EXISTS A FINITESET F



(ASSUMING PZNP)

- THERE IS NO DICHOTOMY FOR S <- ORDERING PROBLEMS
- THMI (G.KUN, J.N. 25)



FEDER VARDI ANALYSIS OF MMSNP 1. (KUN,N. 2008) THREE VERSIONS OF FORBIDDENS LIPTS HAVE FULL POWER IN SHADON $\phi(FORB_{inj}(F'))$ JL FULL POWER OF COLORED UNDIRECTED GRAPHS J1 FULL POWER OF SHADOWS OF ORDERED UNDIRECTED GRAPHS \$ (FORB in 5 (5 4))

FULL POWER OF ϕ (FORB_{inj} (F²)) ψ (LADNER 75) NO DICHOTOMY

LIFT EXPANSION $\mathbf{A}' = (\mathbf{A}, (\mathbf{R}_{Aj} \operatorname{Ret}), (\mathbf{R}_{Aj}' \operatorname{Ret}))$ $\mathbf{A}^{\prime} = (\mathbf{A}_{1}(\mathbf{R}_{\mathbf{A}};\mathbf{R}_{\mathbf{E}}\mathbf{C})_{1} <)$ ORDERED LIFT, EXPANSION $\phi(A') = A = \phi(A^{<})$ REDUCT, FORGETFULL FUNCTOR SHADOW

GRAPHS RELATIONAL STRUCTURES WITH SIGNATURE T $A = (A, (R_A; RET))$

 $\mathcal{F}' = \{ \mathbf{F}'_{A_{j}}, \dots, \mathbf{F}'_{t} \}$ AE \$ (FORBing(5')) J) A J LIFT EXPANSION SUCH THAT FOR ALL i, Asist Fi' \$ A'

J'= { F'_, ... , Fe } $A \in \Phi(FORBinj(S'))$ 1 J LIFT A' SUCH THAT FOR ALL i, Asist Fi' \$ A' ϕ (FORB_{inj} ($F^{<}$)) I JE - ORDERING PROBLEM

15 THERE AN OBSTACLE TO NO DICHOTOMY OF F-ORDERINGS?

15 THERE AN OBSTACLE TO NO DICHOTOMY OF F-ORDERINGS?

NOT CONNECTIVITY



IS THERE AN OBSTACLE TO NO DICHOTOMY OF F-ORDERINGS?

CONNECTIVITY NOT BUT BICONNECTIVITY (2- CONNECTIVITY) THMI LET J BE A FINITE SET BICONNECTED RELAT. STRUCTURES 90 THEN DICHOTOMY HOLDS FOR \$ (FORBinj (5 <))

15 THERE AN OBSTACLE TO NO DICHOTOMY OF F-ORDERINGS

LEMMA FOR EVERY & & AND EVERY B THERE EXISTS B SUCH THAT () **B** -> B 2 B HAS NO CYCLES < L FOR EVERY A ASK (3 IFF ____, A

THE ESSENTIAL PART OF PROOF : SPARSE INCOMPARABILITY

N., X.ZHU 2004 V. MÜLLER 75,79 (SPARSE GRAPHS WITH GIVEN COLOURINGS) - PROTO POLYMORPHISMS N., SIGGERS, ZADORI (FIBER CONSTRUCTION) 2010

89

 $\begin{array}{c} \mathbf{B} \longrightarrow \mathbf{A} \\ \uparrow & / \\ \mathbf{B} \\ \end{array}$

N., RÖDL

FEDER, VARDI, KUN CSP EQUIVALENT TO LARGE GIRTH CSP

MATOUŠEK, N. KUN (2013)

CONSTRUCTIVE PROOFS





VeVB 3 B $\mathbf{G} \rightarrow \mathbf{B}$ \oslash B HAS NO CYCLES 4 L (2)FOR ANY TEMPORAL T (3 B C ---> T

FOR BOUNDED DEGREE OF B LOVA'SZ LOCAL LEMMA + PROBABILISTIC + BALANCING DEGREE & PROB.

(VERIFIES CONJECTURE) OF DUFFUS, GINN AND RÖDL 1995

- BICONNECTED GRAPH F F= COMPLETE.
- NP-COMPLETE FOR EVERY
- F ORDERING PROBLEM IS

ТНМШ

- F² = {F²} SINGLE OBSTRUCTION F BICONNECTED GRAPH
- AN EXACT RESULT

F²- ORDERING PROBLEM VIA INFINITE TEMPORAL TEMPLATE T + BODIRSKY-KARA CHARACTERIZATION OF DICHOTOMY FOR TEMPORAL TEMPLATES

+

PROOF

CONSTRUCTIVE BOUNDED DEGREE SIL

- FULL POWER (META THEOREMS)

- FOR 5 BICONNECTED
- F^C_ ORDERING PROBLEM
- EXACT RESULT FOR

- F[<] - ORDERING PROBLEM FOR TREES



THANK YOU FOR YOUR ATTENTION