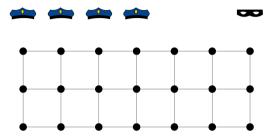
Monotonicity of cops-and-robber-games

Eva Fluck

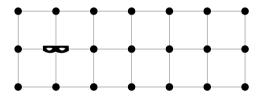
RWTH Aachen University

May 28, 2025

joint work with Isolde Adler and David Philipps

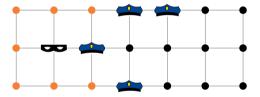






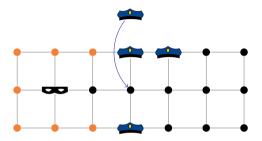
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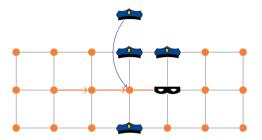
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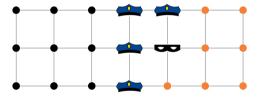
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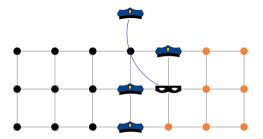
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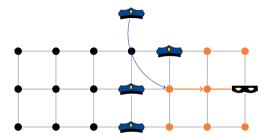
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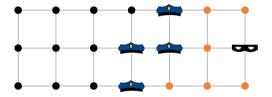
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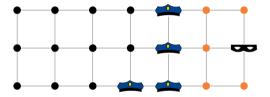
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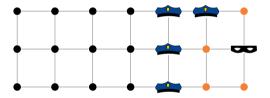
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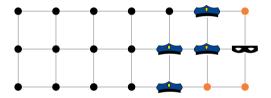
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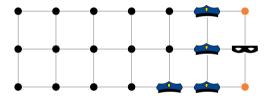
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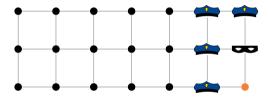
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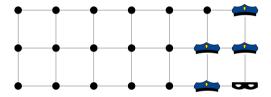
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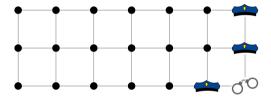
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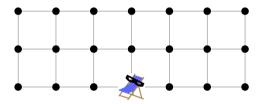


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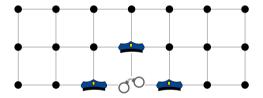
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Cops-and-robber game

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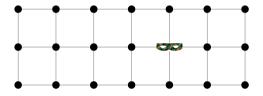
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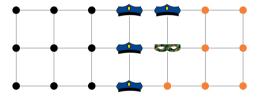
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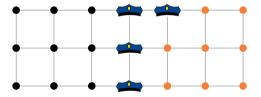
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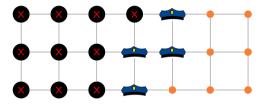
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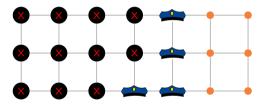
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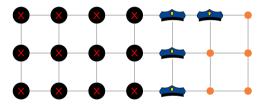
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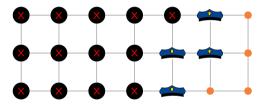
visible lazy

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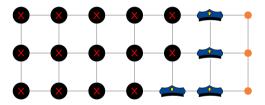
visible lazy

invisible lazy 000



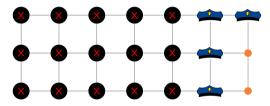
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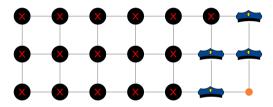
visible lazy

invisible lazy 000



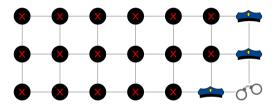
visible lazy

invisible lazy 000



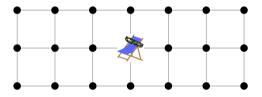
visible lazy

invisible lazy 000



visible lazy

invisible lazy 000



	non-monotone	robber-monotone	cop-monotone
invisible active			
visible active			
œ			
invisible lazy			
X			
visible lazy			
X			

How many cops are needed to catch the robber?

	non-monotone	robber-monotone	cop-monotone
		00	
invisible active	path-width	path-width	path-width
00	[KP85]	[KP86]	[KP86]
visible active	tree-width	tree-width	tree-width
œ	[ST93]	[ST93]	[ST93]
invisible lazy	tree-width	tree-width	
X	[DKT97]	[DKT97]	
visible lazy	∞ -degeneracy		
X	[RT11]		

How many cops are needed to catch the robber?

	non-monotone	robber-monotone	cop-monotone
		50	
invisible active	path-width	path-width	path-width
00	[KP85]	[KP86]	[KP86]
visible active	tree-width	tree-width	tree-width
œ	[ST93]	[ST93]	[ST93]
invisible lazy	tree-width	tree-width	???
X	[DKT97]	[DKT97]	
visible lazy	∞ -degeneracy	???	???
X	[RT11]		

How many cops are needed to catch the robber?

	non-monotone	robber-monotone	cop-monotone
invisible active	path-width	path-width	path-width
00	[KP85]	[KP86]	[KP86]
visible active	tree-width	tree-width	tree-width
	[ST93]	[ST93]	[ST93]
bounded number	???	???	bounded-depth tw
of rounds			[FSS24]
invisible lazy	tree-width	tree-width	???
*	[DKT97]	[DKT97]	
visible lazy	∞ -degeneracy	???	???
X	[RT11]		

	non-monotone	robber-monotone	cop-monotone
		0	8
invisible active	path-width	path-width	path-width
00	[KP85]	[KP86]	[KP86]
visible active	tree-width	tree-width	tree-width
	[ST93]	[ST93]	[ST93]
bounded number	bounded-depth tw	bounded-depth tw	bounded-depth tw
of rounds	[Adler and F., 2024]	[Adler and F., 2024]	[FSS24]
invisible lazy	tree-width	tree-width	???
A Contraction of the second se	[DKT97]	[DKT97]	
visible lazy	∞ -degeneracy	???	???
*	[RT11]		

visible lazy

invisible lazy 000

visible lazy

invisible lazy 000

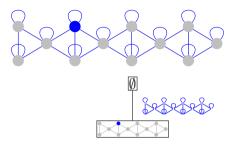
visible lazy

invisible lazy 000

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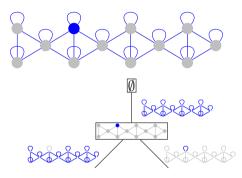
visible lazy

invisible lazy 000



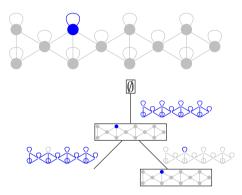
visible lazy

invisible lazy 000



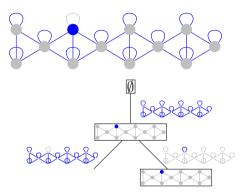
visible lazy

invisible lazy 000



visible lazy

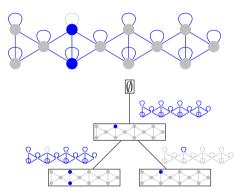
invisible lazy 000



visible active bounded number of rounds $_{O \bullet O O}$

visible lazy

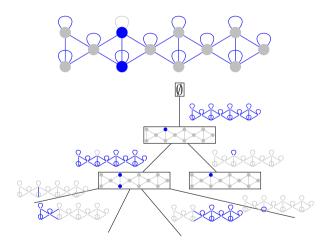
invisible lazy 000



visible active bounded number of rounds $_{\odot \odot \odot \odot}$

visible lazy

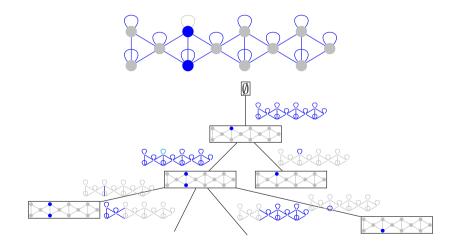
invisible lazy 000



visible active bounded number of rounds $_{\odot \odot \odot \odot}$

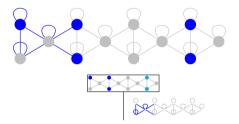
visible lazy

invisible lazy 000



visible lazy

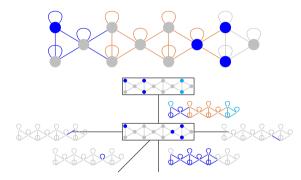
invisible lazy 000



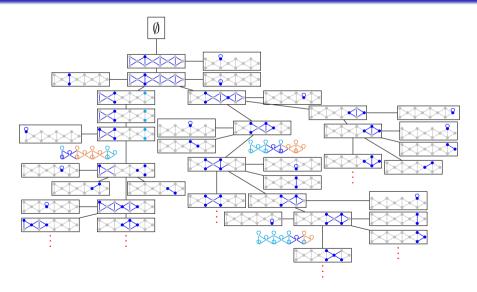
visible active bounded number of rounds ${}_{O \bullet O O}$

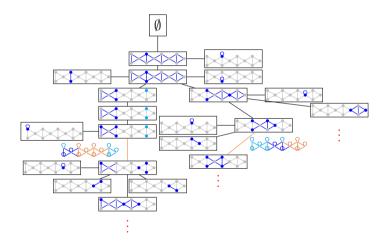
visible lazy

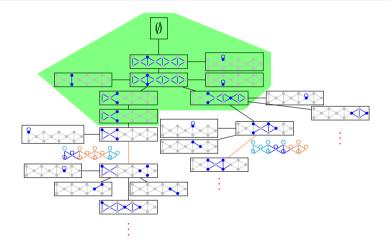
invisible lazy 000

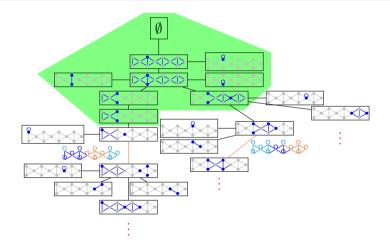


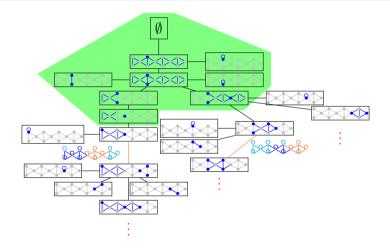
The strategy tree

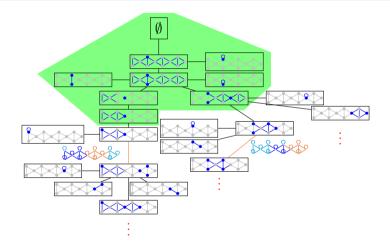


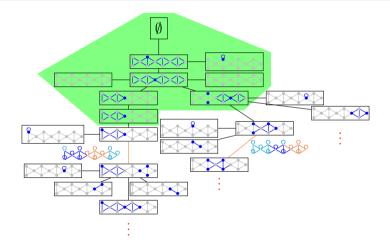


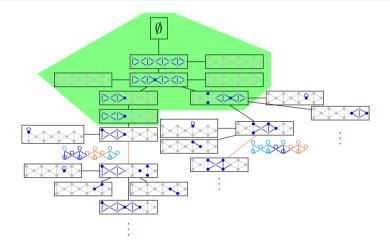


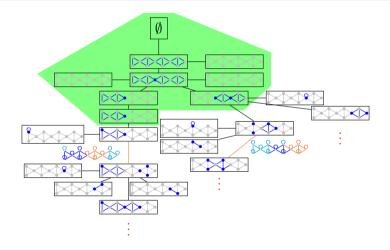


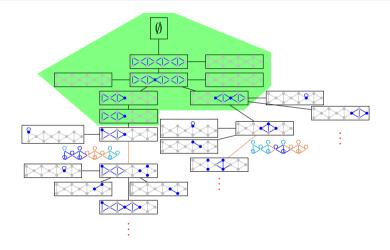


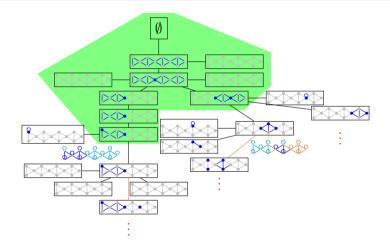












	non-monotone	robber-monotone	cop-monotone
		00	8
invisible active	path-width	path-width	path-width
00	[KP85]	[KP86]	[KP86]
visible active	tree-width	tree-width	tree-width
	[ST93]	[ST93]	[ST93]
bounded number	bounded-depth tw	bounded-depth tw	bounded-depth tw
of rounds	[Adler and F., 2024]	[Adler and F., 2024]	[FSS24]
invisible lazy	tree-width	tree-width	???
X	[DKT97]	[DKT97]	
visible lazy	∞ -degeneracy	???	???
X	[RT11]		

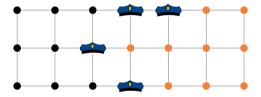
	non-monotone	robber-monotone	cop-monotone
	non-monotone	robber-monotone	cop-monotone
		00	
			×
invisible active	path-width	path-width	path-width
00	[KP85]	[KP86]	[KP86]
visible active	tree-width	tree-width	tree-width
	[ST93]	[ST93]	[ST93]
bounded number	bounded-depth tw	bounded-depth tw	bounded-depth tw
of rounds	[Adler and F., 2024]	[Adler and F., 2024]	[FSS24]
invisible lazy	tree-width	tree-width	???
*	• •		
	[DKT97]	[DKT97]	
visible lazy	∞ -degeneracy	∞ -degeneracy	∞ -degeneracy
×			
44	[RT11]	[F. and Philipps, 2025]	[F. and Philipps, 2025]

	non monotono	robber-monotone	con monotono
	non-monotone	robber-monotone	cop-monotone
		60	
		00	\mathbf{X}
invisible active			noth width
invisible active	path-width	path-width	path-width
	[KP85]	[KP86]	[KP86]
visible active	tree-width	tree-width	tree-width
8	[ST93]	[ST93]	[ST93]
bounded number	bounded-depth tw	bounded-depth tw	bounded-depth tw
of rounds	[Adler and F., 2024]	[Adler and F., 2024]	[FSS24]
invisible lazy	tree-width	tree-width	???
**			
	[DKT97]	[DKT97]	
visible lazy	∞ -degeneracy	∞ -degeneracy	∞ -degeneracy
×		· · · · · · · · · · · · · · · · · · ·	· · · · · · · · · · · · · · · · · · ·
	[RT11]	[F. and Philipps, 2025]	[F. and Philipps, 2025]

	non-monotone	robber-monotone	cop-monotone
		60	
invisible active	path-width	path-width	path-width
00	[KP85]	[KP86]	[KP86]
visible active	tree-width	tree-width	tree-width
	[ST93]	[ST93]	[ST93]
bounded number	bounded-depth tw	bounded-depth tw	bounded-depth tw
of rounds	[Adler and F., 2024]	[Adler and F., 2024]	[FSS24]
invisible lazy	tree-width	tree-width	path-width
X	[DKT97]	[DKT97]	[F. and Philipps, 2025]
visible lazy	∞ -degeneracy	∞ -degeneracy	∞ -degeneracy
2	[RT11]	[F. and Philipps, 2025]	[F. and Philipps, 2025]

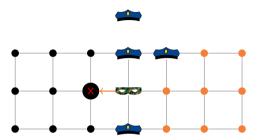
visible lazy

invisible lazy ○●○



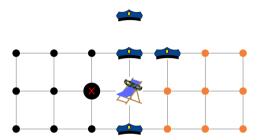
visible lazy

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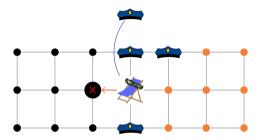
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	non-monotone	robber-monotone	cop-monotone
		60	
invisible active	path-width	path-width	path-width
00	[KP85]	[KP86]	[KP86]
visible active	tree-width	tree-width	tree-width
	[ST93]	[ST93]	[ST93]
bounded number	bounded-depth tw	bounded-depth tw	bounded-depth tw
of rounds	[Adler and F., 2024]	[Adler and F., 2024]	[FSS24]
invisible lazy	tree-width	tree-width	path-width
X	[DKT97]	[DKT97]	[F. and Philipps, 2025]
visible lazy	∞ -degeneracy	∞ -degeneracy	∞ -degeneracy
2	[RT11]	[F. and Philipps, 2025]	[F. and Philipps, 2025]



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